

Control of Mobile Robot Considering Actuator Dynamics with Uncertainties in the Kinematic and Dynamic Models

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Abstract. In this paper, a trajectory tracking control for a nonholonomic mobile robot by the integration of a neural kinematic controller (NKC) and neural dynamic controller (NDC) is investigated, where the wheel actuator (e.g., dc motor) dynamics is integrated with mobile robot dynamics and kinematics so that the actuator input voltages are the control inputs, as well as both the kinematic and dynamic models contains parametric and/or nonparametric uncertainties. The proposed neural controller (PNC) is constituted of the NKC and the NDC, and were designed by use of a modelling technique of Gaussian radial basis function neural networks (RBFNNs). The NKC is applied to compensate the uncertainties in the kinematic parameters of the mobile robot. The NDC, based on the sliding mode theory, is applied to compensate the mobile robot dynamics, and parametric and/or nonparametric uncertainties. Also, the PNC are not dependent of the mobile robot kinematics and dynamics neither require the off-line training process. Stability analysis with basis on Lyapunov theory and numerical simulation is provided to show the effectiveness of the PNC.

Keywords: Mobile robot, trajectory tracking, actuator dynamics, uncertainties, kinematic control, dynamic control, neural networks, Lyapunov theory.

1 Introduction

Several control methods have been proposed for motion control of a mobile robot under nonholonomic constraints [1]-[13]. Some researches consider the dynamics of mobile robot to achieve ‘perfect velocity tracking’ [1], [7]. However, in these methods the perfect knowledge about the parameter values of mobile robot is necessary. Usually, such requirement is unattainable. In practical situations, obtaining exact parameter values on a mobile robot is almost impossible. There are some results on the problems, regarding the integration of both, kinematic and neural dynamic controllers for a mobile robot with uncertainties and/or disturbances in the dynamics [2], [4], [6], [8], [10], [11]. These results are based on the assumption that the kinematics of the system is exactly known or by selecting a special control target (i.e. the forward

velocity and the angular velocity of the robot) to avoid this problem. However, in practice, there are uncertainties in both kinematics and dynamics.

As there are few works [3], [5], [9] dealing of both the unknown dynamics and unknown kinematics, the PNC that addresses the problem of integration of the NKC and the NDC (based on the sliding mode theory), considering the presence of parametric and/or nonparametric uncertainties in the kinematic and dynamic models.

In this paper, the wheel actuator (e.g., dc motor) dynamics is integrated with mobile robot dynamics and kinematics so that the actuator input voltages are the control inputs (is more realistic), and differently from other investigations with neural networks in the control of mobile robots [2], [4], [6], [8], [10], [11], the others contributions are: the implementation of the PNC (NKC plus NDC) based on the partitioning of the RBFNN into several smaller subnets in order to obtain more efficient computation; the PNC neither require the knowledge of the mobile robot kinematics and dynamics nor the time-consuming training process; the stability analysis and convergence of the mobile robot control system, and the learning algorithms are proved by using Lyapunov theory.

2 Kinematics and Dynamics of a Mobile Robot

The mobile robot's position (Figure 1) is given by the posture vector and $q = [x_c \ y_c \ \theta]^T$, containing the center of mass (guidance point) C coordinates and the heading angle θ , with P , d , r , and $2R$ being intersection of the axis of symmetry with the driving wheel axis, distance from the point C to the point P , radius of wheels, and distance between wheels, respectively.

Without considering the effect of surface friction $\bar{F}(\dot{q})$ and gravitational torques $\bar{G}(q) = 0$, the dynamic equations of the nonholonomic mobile robot for control purposes are:

$$\dot{q} = S(q)v, \quad \bar{H}(q)\dot{v} + \bar{C}(q, \dot{q})v + \bar{\tau}_d = \bar{\tau}, \quad (1)$$

where the properties are maintained, as well as matrices, vectors, and variables are defined as in [2].

Neglecting motor inductance in the electrical part of the actuator [12], the equations governing the actuator motor can be written as:

$$\tau_m = K_T i, \quad u = R_a i + K_b \dot{\phi}_m, \quad (2)$$

where τ_m is the torque generated by the motor, K_T is the motor torque constant, i is the current, u is the actuator input voltage, R_a is the resistance, K_b is the counter electromotive force coefficient, and $\dot{\phi}_m$ is the velocity of the actuator motor.

The angular velocity of the actuator motor, $\dot{\phi}_m$, and the corresponding wheel angular velocity v are related by gear ratio N as

$$v = \dot{\phi}_m / N, \quad (3)$$

and the motor torque τ_m is related to the wheel torque τ as:

$$\tau = N\tau_m. \quad (4)$$

Using (2)-(4), the mobile robot dynamics equation (including actuator dynamics) can be written as:

$$\bar{H}(q)\dot{v} + \bar{C}(q, \dot{q})v + \bar{\tau}_d = (NK_T/R_a)\bar{B}u - (N^2K_TK_b/R_a)\bar{B}v = T. \tag{5}$$

From (1), the posture kinematic model of a mobile robot with differential drive can be represented in the cartesian coordinates system, in the matricial form, as:

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r}{2R}(R \cos \theta - d \sin \theta) & \frac{r}{2R}(R \cos \theta + d \sin \theta) \\ \frac{r}{2R}(R \sin \theta + d \cos \theta) & \frac{r}{2R}(R \sin \theta - d \cos \theta) \\ \frac{r}{2R} & -\frac{r}{2R} \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix}, \tag{6}$$

where $v = [\omega_r \ \omega_l]^T$ is angular velocity of the wheels. This kinematic model (8) is obtained from the description of the angular velocity of the wheels in function of the linear and angular velocities (v_L, ω_A) of the mobile robot through the following relationship:

$$\begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} = \begin{bmatrix} 1/r & R/r \\ 1/r & -R/r \end{bmatrix} \frac{\tan \theta \cot \theta}{D(q)} \begin{bmatrix} v_L \\ \omega_A \end{bmatrix}. \tag{7}$$

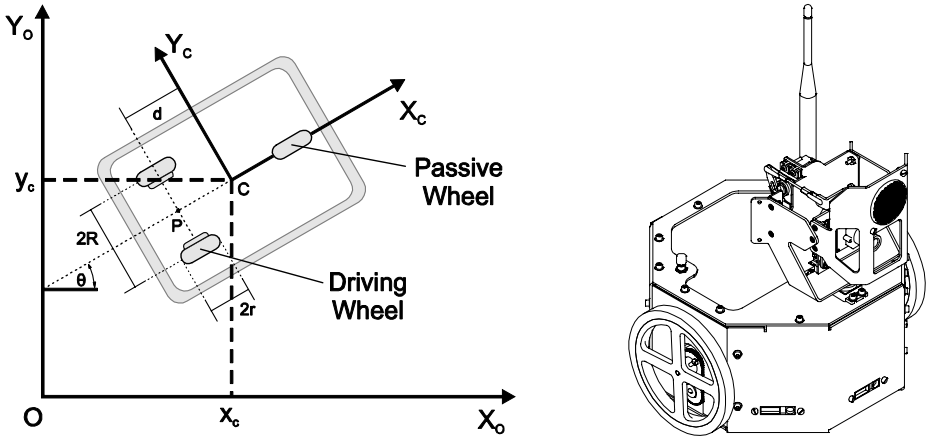


Fig. 1. Model of a nonholonomic mobile robot

3 Neural Networks Modelling by RBFNNs

Based on (5), it can be verified that $\bar{H}(q)$ is function of q only, and $\bar{C}(q, \dot{q}) = \bar{C}(z)$ is a function of q and \dot{q} , thus, static neural networks and dynamic neural networks are enough to model them, respectively. Therefore, the stability of the neural networks can be analyzed, where matrix Ge-Lee (GL) [13], defined by $\{.\}$, and its product operator ' \bullet ' are used. The ordinary matrix and vector are denoted by $[.]$.

Foregrounded in (5), the matrices $\bar{H}(q)$ and $\bar{C}(z)$ of the mobile robot dynamics can be expressed by:

$$\bar{H}(q) = [\{W_{\bar{H}}\}^T \cdot \{\xi_{\bar{H}}(q)\}] + E_{\bar{H}}(q), \quad \bar{C}(z) = [\{W_{\bar{C}}\}^T \cdot \{\xi_{\bar{C}}(z)\}] + E_{\bar{C}}(z), \quad (8)$$

where $\{W_{\bar{H}}\}$, $\{\xi_{\bar{H}}(q)\}$, $\{W_{\bar{C}}\}$, and $\{\xi_{\bar{C}}(z)\}$ are GL matrices, and their respective elements are $W_{\bar{h}_{kj}}$, $\xi_{\bar{h}_{kj}}(q)$, $W_{\bar{c}_{kj}}$, and $\xi_{\bar{c}_{kj}}(z)$. $E_{\bar{H}}(q) \in R^{n \times n}$ and $E_{\bar{C}}(z) \in R^{n \times n}$ are matrices, and their modelling error elements $\varepsilon_{\bar{h}_{kj}}(q)$ and $\varepsilon_{\bar{c}_{kj}}(z)$, respectively.

It is important to emphasize that a $P(\cdot)$ vector can be modeled with static neural networks, since it is a function of a variable only. Thus, $P(\cdot)$ results in:

$$P(\cdot) = [\{W_p\}^T \cdot \{\xi_p(\cdot)\}] + E_p(\cdot), \quad (9)$$

where $\{W_p\}$ and $\{\xi_p(\cdot)\}$ are GL vectors, and their respective elements are W_{p_k} and $\xi_{p_k}(\cdot)$. $E_p(\cdot) \in R^n$ is a vector, and their modelling error elements $\varepsilon_{p_k}(\cdot)$.

4 Neural Control of the Kinematic Model

Let v_s be a selected smooth velocity input, that achieves tracking for (6), given by:

$$v_s = \begin{bmatrix} v_f \\ \omega_f \end{bmatrix} = \begin{bmatrix} v_d \cos e_3 + k_1 e_1 \\ \omega_d + k_2 v_d e_2 + k_3 v_d \sin e_3 \end{bmatrix}, \quad (10)$$

where $v_d > 0$ for all t is the reference linear velocity; ω_d is the reference angular velocity; k_1 , k_2 , and k_3 are positive parameters; and $e_q = [e_1 \ e_2 \ e_3]^T$ is the position tracking error vector [14].

From kinematics, (6), if the parameters, r , R , and d are unknown, it is possible to use the estimates of these parameters in (7) and design learning algorithms for the neural controller to compensate these parameters. Assume:

$$a_1(q) = \frac{d}{rd} D(q) = \frac{1}{r} D(q), \quad b_1(q) = \frac{Rd}{rd} D(q) = \frac{R}{r} D(q). \quad (11)$$

Replacing the linear and angular velocities in (7) with (10), and using (9) with $E_p(\cdot) = 0$, assume that the $\hat{A}(q)$ and $\hat{B}(q)$ vectors of (12) can be modeled with static neural networks as:

$$v_c = \begin{bmatrix} v_{c1} \\ v_{c2} \end{bmatrix} = \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} = \underbrace{\begin{bmatrix} \hat{a}_1(q) \\ \hat{a}_1(q) \end{bmatrix}}_{\hat{A}(q)} v_f + \underbrace{\begin{bmatrix} \hat{b}_1(q) \\ -\hat{b}_1(q) \end{bmatrix}}_{\hat{B}(q)} \omega_f + \underbrace{\begin{bmatrix} \hat{W}_{A1}^T \xi_{A1}(q) \\ \hat{W}_{A2}^T \xi_{A2}(q) \end{bmatrix}}_{\hat{A}(q)} v_f + \underbrace{\begin{bmatrix} \hat{W}_{B1}^T \xi_{B1}(q) \\ \hat{W}_{B2}^T \xi_{B2}(q) \end{bmatrix}}_{\hat{B}(q)} \omega_f, \quad (12)$$

where $\hat{a}_1(q)$ and $\hat{b}_1(q)$ are the estimates of $a_1(q)$ and $b_1(q)$; $\tilde{a}_1(q) = a_1(q) - \hat{a}_1(q)$ and $\tilde{b}_1(q) = b_1(q) - \hat{b}_1(q)$ are the estimated errors respectively; $\hat{W}_{A1}^T \xi_{A1}(q) = \hat{W}_{A2}^T \xi_{A2}(q)$, $\hat{W}_{B2}^T \xi_{B2}(q) = -\hat{W}_{B1}^T \xi_{B1}(q)$, and the wheel angular velocity is defined as the velocity control input v_c .

To design the actuator input voltages, one defines the auxiliary velocity tracking error as:

$$e_c = v_c - v = \begin{bmatrix} v_{c1} - \omega_r \\ v_{c2} - \omega_l \end{bmatrix}. \quad (13)$$

Consider the Lyapunov function candidate in the form:

$$V_1 = \frac{1}{2} \left(e_1^2 + e_2^2 + 2 \frac{(1 - \cos e_3)}{k_2} + \frac{(\hat{W}_{A1}^T \xi_{A1}(q))^2}{\gamma_1 W_{A1}^T \xi_{A1}(q)} + \frac{(\hat{W}_{B1}^T \xi_{B1}(q))^2}{\gamma_2 W_{B1}^T \xi_{B1}(q)} \right), \quad (14)$$

with $\gamma_1 > 0, \gamma_2 > 0$.

The parameter learning rules are defined as:

$$\hat{W}_{A1} = \gamma_1 e_1 v_f (\xi_{A1}^+(q))^T, \quad \hat{W}_B = \frac{\gamma_2 \sin e_3}{k_2} \omega_f (\xi_{B1}^+(q))^T, \quad (15)$$

where $\xi^+(\cdot)$ is a pseudoinverse matrix.

Performing mathematical manipulations [14], \dot{V}_1 becomes:

$$\dot{V}_1 = -k_1 e_1^2 - \frac{k_3 v_d \sin^2 e_3}{k_2}. \quad (16)$$

Apparently, $\dot{V}_1 \leq 0, \dot{V}_1 = 0$ if only if, $e_1 = 0$ and $e_3 = 0$. Moreover, (10) and (12) show that $v_{c1} = \omega_r$ and $v_{c2} = \omega_l$ are bounded. More details can be found in [14].

5 Neural Control of the Dynamic Model

Let Λ be a symmetric diagonal positive definite matrix, one defines:

$$\begin{aligned} v_r &= v_c + \Lambda \int_0^t e_c dt, & \dot{v}_r &= \dot{v}_c + \Lambda e_c, \\ s &= v_r - v = e_c + \Lambda \int_0^t e_c dt, & \dot{s} &= \dot{v}_r - \dot{v} = \dot{e}_c + \Lambda e_c, \end{aligned} \quad (17)$$

where s is a filtered tracking error term, $\int_0^t e_c dt$ is an auxiliary position tracking error, which does not reflect the position tracking error e_q directly, besides not having physical meaning.

One defines the control input to be of the form:

$$\begin{aligned} u &= (R_a / NK_T) \bar{B}^{-1} \left([\{\hat{W}_H\}^T \cdot \{\xi_H(q)\}] \dot{v}_r + [\{\hat{W}_C\}^T \cdot \{\xi_C(z)\}] v_r + \rho - \gamma \right), \\ \rho &= (N^2 K_T K_b / R_a) \bar{B} v_r, \quad \gamma = -G \tanh(\delta s) - (Q + I_n) s, \end{aligned} \quad (18)$$

where $\{\hat{W}_H\}$, and $\{\hat{W}_C\}$ represent estimates of true parameters of matrices $\{W_H\}$, and $\{W_C\}$ of (8), and γ is the robustifying term with the aim of compensating for the bounded unknown disturbances, with $G^T = G > 0, \delta > 0, (Q + I_n)^T = (Q + I_n) > 0$, and I_n is identity matrix. The $\tanh(\cdot)$ function is adopted for the purpose of deriving closed-loop stability.

Let us consider Lyapunov function candidate:

$$\begin{aligned} V_2 &= \frac{1}{2} \left(s^T \bar{H}(q) s + \sum_{k=1}^n \tilde{W}_{Hk}^T \Gamma_{Hk}^{-1} \tilde{W}_{Hk} + \sum_{k=1}^n \tilde{W}_{Ck}^T \Gamma_{Ck}^{-1} \tilde{W}_{Ck} \right) + \\ &\quad + \left(\int_0^t e_c dt \right)^T \Lambda \int_0^t e_c dt, \end{aligned} \quad (19)$$

being Γ_k are dimensional compatible symmetric positive definite matrices, and $\{\tilde{W}_k\} = \{W_k\} - \{\hat{W}_k\}$ is parameter error. Clearly, $V_2 \geq 0$, and $V_2 = 0$ if only if $e_c = 0$, $\int_0^t e_c dt = 0$, $s = 0$, and $\{\tilde{W}\} = 0$.

The parameter learning laws of neural networks are chosen as:

$$\begin{aligned}\dot{\hat{W}}_{\bar{H}_k} &= \Gamma_{\bar{H}_k} \bullet \{\xi_{\bar{H}_k}(q)\} \dot{v}_r s_k - K_{\bar{H}_k} \Gamma_{\bar{H}_k} \|s\| \hat{W}_{\bar{H}_k}, \\ \dot{\hat{W}}_{\bar{C}_k} &= \Gamma_{\bar{C}_k} \bullet \{\xi_{\bar{C}_k}(z)\} \dot{v}_r s_k - K_{\bar{C}_k} \Gamma_{\bar{C}_k} \|s\| \hat{W}_{\bar{C}_k},\end{aligned}\quad (20)$$

with $K_k = K, > 0$ are positive constants.

After the necessary mathematical manipulations [15], \dot{V}_2 stays:

$$\begin{aligned}\dot{V}_2 \leq & -\|e_c\|^2 - \|s\|^2 Q_{\min} - (N^2 K_T K_b / R_a) s^T \bar{B} s - \\ & - \|s\| (G_{\min} \|\tanh(\delta s)\| - (b_d + e_{NN} + \chi)) - \left(\int_0^t e_c dt\right)^T \Lambda^T \Lambda \int_0^t e_c dt,\end{aligned}\quad (21)$$

with Q_{\min} , and G_{\min} are the minimum singular values of Q and G , respectively. Because of $\|\tanh(\delta s)\| > \frac{b_d + e_{NN} + \chi}{G_{\min}}$, \dot{V}_2 is guaranteed negative.

To ensure that the global system is stable, the Lyapunov function candidate is given as $V = V_1 + V_2$, where V_1 and V_2 refers to (14) and (19) respectively. Since \dot{V}_1 in (16) and \dot{V}_2 in (21) are guaranteed to be negative, then \dot{V} is also guaranteed negative. More details can be found in [15].

6 Simulations Results

In the realization of the simulations, the kinematic and the dynamic (including actuator dynamics) models described in [16] are used. The model parameters of the prototype wheeled mobile robot estimated in [17] are: $m = 11.0$ kg, $I = 1.057$ kgm², $R = 0.265$ m, $r = 0.125$ m, $d = 0.1$ m, $N = 21$, $K_T = [0.057 \ 0.051]^T$ Vs, $K_b = [0.057 \ 0.051]^T$ Vs, and $R_a = [0.476 \ 0.233]^T \Omega$. The reference trajectory is a straight line with initial coordinates $[x_r(0), y_r(0), \theta_r(0)] = [1, 2, 25.56^\circ]$. The initial position of the robot is $[x_c(0), y_c(0), \theta_c(0)] = [2, 1, 10^\circ]$. The parameters of the NKC are chosen as $k_1 = 1$, $k_2 = 3$, $k_3 = 7$, $\gamma_1 = 35$, $\gamma_2 = 5$, and $\sigma = 7$; and the gains of the NDC as $\Lambda = \text{diag}[2]$, $\Gamma_k = 0.1$, $\sigma = 3$, $K = 0.001$, $G = \text{diag}[1]$, $Q = \text{diag}[1]$.

A Coulomb friction and a bounded periodic disturbance term are added to the robot system as:

$$\bar{\tau}_d = [(f_1 + f_1(t)) \text{sgn}(\omega_r) + 0.1 \sin 2\pi t \quad (f_2 + f_2(t)) \text{sgn}(\omega_l) + 0.1 \cos 2\pi t]^T, \quad (22)$$

where $f_1 = 1.0$, $f_2 = 1.0$. Function $f(t)$ is nonlinear, defined as: $[f_1(t) \ f_2(t)] = [0.0 \ 0.0]^T$ if $t < 3$; $[f_1(t) \ f_2(t)] = [0.0 \ 2.0]^T$ if $3 \leq t \leq 8$; $[f_1(t) \ f_2(t)] = [2.0 \ 2.0]^T$ if $t \geq 8$, respectively. Thus, $\bar{\tau}_d$ is subject to a sudden change at time goes to 3s and 8s. Moreover, in 8s, the mobile robot suddenly dropped of an object of 30.0 kg, that is, a quarter of its original mass.

The tracking performance of the PNC can be observed in the: Figure 2, since the mobile robot naturally describes a smooth path tracking over the desired trajectory and that the tracking errors tend to zero; Figure 3 that the robot velocities tend to desired values and that shows the behavior of the wheel actuator input voltages.

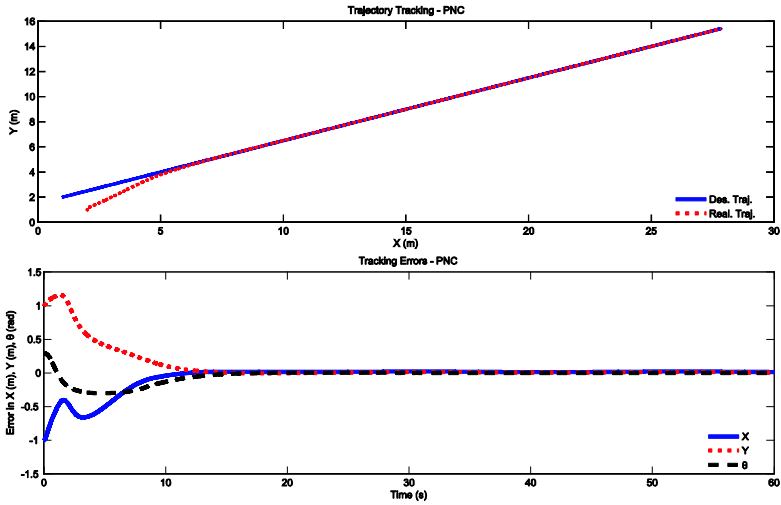


Fig. 2. PNC - Trajectory tracking and tracking errors

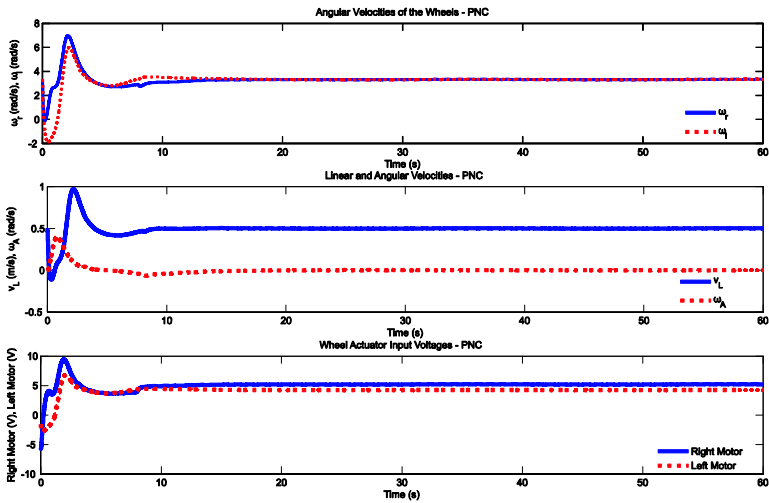


Fig. 3. PNC - Velocities and wheel actuator input voltages

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